

Student Error in Grade X on the Concept of Trigonometry Using a Hermeneutic Phenomenological Study

Alifia Qolbiyatus Syifa¹⁾, Ira Kurniawati¹⁾ *

¹⁾Department of Mathematics Education, Universitas Sebelas Maret. Surakarta, Indonesia.

Abstract: Trigonometric Comparison is an E-phase mathematics material of class X geometry elements in the Merdeka Curriculum. This study aims to identify students' difficulties and learning obstacles in understanding the material. This research employs a qualitative study design using a hermeneutic phenomenological approach, focusing on grade X students and their math teacher as subjects. Data were collected through written tests (TKR), documentation, and interviews. Then it was analyzed using Ricoeur's stages. The research sample was selected purposively based on the meaning category of students' answers to TKR questions, which were grouped into four categories. The results indicated that students had difficulty in choosing the right trigonometric ratio and understanding the context of applied problems. Furthermore, the findings reveal three categories of learning obstacles as proposed by Brousseau, which students encounter: ontogenic obstacles of a psychological nature, indicated by students' low interest and motivation in learning; instrumental obstacles, reflected in the imbalance between students' use of concept images and concept definitions; and conceptual obstacles, due to students' lack of mastery of prerequisite knowledge. In addition, epistemological obstacles were identified, as evidenced by students' difficulties in solving problems presented in various forms, and didactical obstacles, as demonstrated by the teachers' rapid and superficial delivery of the material. These findings suggest that students' learning obstacles are interconnected between internal factors (within the students) and external factors.

Keywords: learning obstacle; phenomenology hermeneutic; trigonometry ratio.

INTRODUCTION

Trigonometry is a section of mathematics that serves as a basis for studying more advanced mathematical topics. This material is taught at all levels of school because the role of trigonometry is not only theoretical but also a tool for understanding and solving problems in several real-world contexts, such as calculating the height or width of an object without direct measurement (Dhungana, Pant & Dahal, 2023). Based on this, the application of trigonometry in everyday life is very much. Therefore, high school students must master the basic concepts of trigonometry well, as this material forms the foundation for understanding other fields of mathematics and science.

The mathematics learning process is complex and interrelated, as it involves not only students but also the interaction among students, teachers, and mathematics itself (La Misi, Saidi & Tamrin Bakar, 2023). In the Merdeka Curriculum, the trigonometric comparison is one of the mathematics materials in phase E of class X at the SMA/MA level in the geometry element. Trigonometry material is quite tricky for students, as revealed by (Thasiro & Ruzai, 2023). The material, which is quite challenging, requires a learning process that is carried out in stages, starting with the introduction of basic concepts and progressing to more complex ones (Sukwantini, 2020). The independent curriculum offers more flexibility for teachers but also requires them to be creative in designing innovative learning activities, allowing students to develop a greater appreciation for the material.

The learning objectives in trigonometric comparison material are for students to understand the relationship between angles and sides in right triangles and to be able to apply the concept of trigonometric comparison in solving contextual problems. However, the results of daily tests on trigonometric comparison material show that out of 36 students in class X, 19

* Correspondence to Author. E-mail: irakurniawati@staff.uns.ac.id

Publisher: Department Mathematics Education, Universitas Halu Oleo

students still have scores below the Learning Objective Completeness Criteria (KKTP) and students still have difficulty in determining the value of trigonometric comparisons (sin, cos, and tan), mistakenly stating the value of trigonometric comparisons for special angles, and difficulty in solving contextual problems. (Fajri & Nida, 2019) also said that as many as 80.95% of students still have difficulty in defining trigonometric concepts (sine, cosine, and tangent); 40.86% of students have difficulty understanding principles where students have difficulty performing arithmetic operations and cannot determine the value of comparisons at special angles; and as many as 38.1% of students have difficulty solving problems in verbal form where students are unable to show known elements in narrative issues. These difficulties indicate issues related to the interpretation of trigonometric comparisons and their significance, which are closely tied to students' learning experiences.

Phenomenology, according to (Creswell, 2013) is a study in which a person's experience will be described, and the meaning of the experience will be understood. Meanwhile, Van Manen (Creswell, 2013) defines hermeneutics as a philosophy or approach to interpreting the meaning of text or experience. Hermeneutic phenomenology enables a more profound understanding of how students experience and impart meaning to the mathematical concepts they learn, including the learning difficulties or obstacles they encounter during the learning process. (Jatisunda, Suciawati & Nahdi, 2021) explained that students have a relatively poor concept image of the Pythagorean theorem, especially when determining the length of a side of a right triangle. Tall and Vinner (Jatisunda, Suciawati & Nahdi, 2021) state that a concept image describes the overall cognitive structure related to the concept and includes all mental images, properties, and processes related to and built over many years through various experiences (phenomenology), which can change when an individual meets new stimuli or experiences. Tall and Vinner (Jatisunda, Suciawati & Nahdi, 2021) also define "concept definition" as the language used by each individual to describe a particular concept. This definition is conveyed in language that is easy to understand, either through the everyday language used by the teacher or with words familiar to students.

In mathematics learning, meaning construction is a crucial process through which students develop an understanding of concepts based on their learning experiences. Students receive information from the teacher and actively form their understanding. This process is influenced by numerous factors, resulting in each student having a unique understanding shaped by their individual learning experience (La Misi, Saidi & Tamrin Bakar, 2023). In the learning process, students often use concept images and concept definitions. Vinner (Nurwahyu, Tinungki & Mustangin, 2020) stated that the simultaneous use of concept images and concept definitions could lead to students' incompleteness in using both. This imbalance has the potential to cause learning obstacles.

Learning obstacles in mathematics can generally be interpreted as obstacles to thinking and understanding mathematics, such as theories, concepts, and other related issues. According to (Brousseau, 2002), learning obstacles are a piece or slice of knowledge, not a lack of it, received by students. Therefore, students who experience learning obstacles or difficulties cannot be said to have encountered learning obstacles if they have not yet acquired this knowledge. (Brousseau, 2002), categorizes learning obstacles into three types. First, ontogenic obstacles, which are learning obstacles related to the students themselves. Second, epistemological obstacles arise from difficulties in the learning process due to the limited knowledge and understanding that students possess. The third learning obstacle is a didactical obstacle, which arises from issues within the didactical system. This study aims to determine students' difficulties in trigonometric comparison material and identify learning obstacles (ontogenic, epistemologic, and didactic obstacles) in trigonometric comparison material based on students' meaning and learning experience.

METHODS

This study is qualitative research, where qualitative methods utilize natural contexts to understand phenomena (Denzin & Lincoln, 2018). The hermeneutic phenomenological approach is used in this study because it allows researchers to understand students' subjective experiences in learning more deeply. A qualitative research method with a hermeneutic phenomenology approach is used to describe and interpret the learning experience and the meaning students give to the experience. In this study, the results of hermeneutic phenomenology can identify and categorize learning obstacles that occur to students.

The data in this study include the results of a written test, namely the Respondent Ability Test (TKR), interviews, and documentation. Subjects were selected based on the category of answers given by students on the Respondent Ability Test (TKR) regarding the meaning of trigonometric comparison. The data analysis steps used in this study adopted the hermeneutic phenomenology approach according to Ricoeur (Ghasemi et al, 2011) which focuses on the interpretation of meaning as follows: (a) Explanation. At this stage, the data that has been obtained will be recapitulated; (b) Naïve Understanding. After recapitulating the obtained data, the next step is to understand its initial meaning by creating textural descriptions (what students experience) and structural descriptions (how these experiences are formed) based on the answers expressed by students; (c) In-depth Understanding. This step involves analyzing the connection between the textural description and the structural description and creating a combined description based on the results of the previous analysis; (d) Appropriation. At this stage, the analysis focuses on the relationship between the combined description and significant statements made by the mathematics teacher, as well as relevant data sources or theories.

RESULTS AND DISCUSSION

To obtain data in achieving these objectives, research instruments are used, namely the Respondent Ability Test (TKR), interviews, and documentation. TKR consists of seven description questions given to 35 students of class X with a processing time of 45 minutes. This Respondent Ability Test (TKR) aims to be a tool to identify learning obstacles experienced by students in understanding the basic concepts of trigonometric comparison. Interviews were conducted with four selected students and mathematics teachers to obtain perspectives related to the meaning of trigonometric comparison and its influence on the learning process in the classroom.

The meaning of "trigonometric comparison" refers to the definition or understanding of trigonometric comparison. The categories of the meaning of trigonometric comparison answers expressed by students based on the TKR results are divided into four, namely: (a) trigonometric comparison as the comparison between the sides of a right-angled triangle relating to a specific angle (T1); (b) meaning of trigonometric comparison as comparison of triangle side lengths (T2); (c) meaning of trigonometric comparison as comparison of side lengths of flat buildings (T3); (d) other meaning findings related to trigonometric comparison (T4).

• Difficulty in Selecting and Applying Appropriate Trigonometric Ratios

One of the difficulties experienced by students in understanding trigonometric comparison material is the inaccuracy in choosing and applying trigonometric comparisons that are appropriate to the context of the problem. Figure 1 below is Subject S2's answer to the TKR question.

The following illustration shows a part of the roof of a building. If the distance from the ridge of the roof to the ceiling is 1.2 m and the slope of the roof is 45° , determine the width of the building (l).

Ilustrasi berikut menunjukkan bagian atap dari sebuah bangunan. Jika jarak bubung atap ke langit-langit adalah 1,2 m dan kemiringan atap tersebut adalah 45° , tentukan lebar bangunan tersebut (l).



Jawaban:

Cari lebar

$$\sin 45^\circ = \frac{de}{m_1}$$

$$\frac{1}{2}\sqrt{2} = \frac{1,2}{m_1}$$

$$m_1 = \frac{1,2}{\frac{1}{2}\sqrt{2}}$$

$$m_1 = 1,2 \cdot \sqrt{2}$$

$$= 2,4\sqrt{2}$$

Figure 1. Subject S2's Answer to Problem Number 4

In Figure 1, it can be seen that Subject S2 was wrong in using the appropriate trigonometric comparison, where the comparison used should be $\tan 45^\circ$, but Subject S2 used $\sin 45^\circ$. This was also revealed by Subject S2 in the interview. The following is an excerpt of the interview with Subject S2.

- P : What about number four?
- S2 : Number four is looking for the width, then the angle is known 45° and the height is 1.2 m. Then use sin to find the width. The sin formula is front per slope, meaning that $\sin 45^\circ$ is half the root of two, so it means this. After that, multiply it by two so that it becomes 2.4 roots of two
- P : Wich width do you think it is?
- S2 : This one (pointing to the picture of the sloping side of the roof)
- P : That one?
- S2 : Yes
- P : Isn't it already in the question, which width is it?
- S2 : Eumm, oiya there are letters L
- P : Do you have trouble deciding which formula to use?
- S2 : Yes, I was confused about using sin or tan

Difficulty in determining the appropriate trigonometric ratio occurs when students cannot identify the relationship between the sides in a right triangle to the given angle. This is in line with the results of research conducted by (Susanti et al., 2024) which states that students experience many mistakes because they cannot understand the concept of the sine and cosine rules correctly because of the selected sides.

- Difficulty in Understanding the Context of Applied Trigonometric Comparison Problems

In addition to difficulties in choosing the right trigonometric comparison, students also have difficulty in understanding the context of applied problems involving trigonometric comparison. In Figure 2 below is the answer number 6 given by Subject S2 on the Respondent Ability Tes (TKR).

Sebuah kapal akan berlayar ke arah timur. Dari pelabuhan, kapal tersebut melihat puncak mercusuar pada arah 30° . Setelah berlayar sejauh 200 meter, kapal tersebut melihat mercusuar yang sama pada arah 60° . Tentukan tinggi dari mercusuar tersebut.

Jawaban:

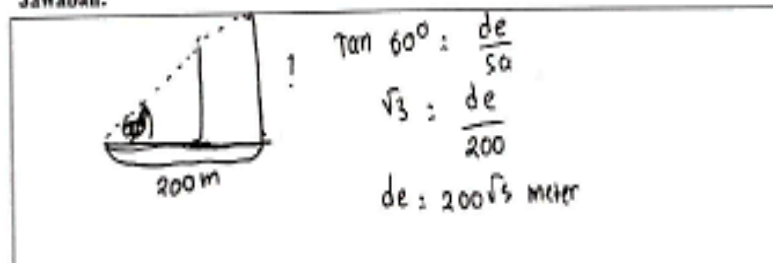


Figure 2. Subject S2's Answer to Problem Number 6

In Figure 2, it can be seen that Subject S2 did not compile a complete mathematical model according to the information in the problem. The drawing made only shows one right triangle, which should show two right triangles to show the results of two different observation positions. As a result, the solution did not produce the right answer. This difficulty was also expressed by other students through interviews conducted. The following are interview excerpts with Subjects S2, S3, and S4.

- P : Okay, which of the materials taught and this question is the most difficult?
- S2 : If the most difficult of the questions is number six
- P : Oh yes, from this material, or from the problems that were done the most difficult and which do you think is difficult?
- S : The difficult one is number six, I find it a bit difficult to do it so I work with my friends.
- P : ... Are these questions easy or difficult?
- S3 : Not hard enough
- P : Which one is difficult?
- S3 : Those are the story problems, especially number six.
- P : Oh yes, from this material, or from the problems that were done the most difficult and which do you think is difficult?
- S4 : I think number six is the hard one, because it has two angles. But the last three numbers are the ones that you have to think about doing first.

Interviews with students revealed that problem number 6 on the Respondent Ability Test (TKR), which involves applying trigonometric comparisons to elevation angles, was considered the most difficult. Students experienced difficulties with applied problems, specifically their inability to visualize real situations presented in the Respondent Ability Test (TKR) questions and to connect the information contained in those problems. This is in line with research conducted by (Maharani & Prihatnani, 2019) which states that the inability to visualize invisible space can trigger errors in solving geometry problems.

- Student Learning Obstacle on Trigonometric Comparison Material

The meaning of trigonometric comparison according to students and students' learning experiences in interpreting the basic concepts of trigonometric comparison have been described. However, this cannot be a guarantee that students have understood trigonometric comparison. Therefore, it is necessary to explore the existence of learning obstacles or learning barriers experienced by students in trigonometric comparison. Learning obstacles are

divided into three categories, namely ontogenic obstacles, epistemological obstacles, and didactical obstacles.

The first, ontogenic obstacles, are barriers that arise in students due to the limited stage of cognitive development (Brousseau, 2002). This obstacle is linked to students' learning abilities and cognitive development, which are insufficient for thoroughly understanding the material. Ontogenic obstacles are divided into three main categories as expressed by Brousseau in (Pauji et al., 2023), namely psychological factors such as interest and motivation, because the interest and motivation that students have will affect the level of understanding of the material obtained and determine the possibility of errors made; instrumental factors such as technical matters of learning, namely errors that occur in the completion of tasks by students; and conceptual factors such as mastery of prerequisite materials and basic concepts. Based on the results of the interview, it was found that there were students who had a low level of interest and motivation in trigonometric comparison material. Interviews related to student interest and motivation were conducted because the interest and motivation students have will affect the understanding gained and the mistakes that students may make. These findings can be observed in some of the following interview excerpts.

- P : Oh I see, but do you have any interest in learning math?
 S4 : A little bit
 P : Why a little?
 S4 : Yes, I was lazy, I didn't want to go to school here at first.
 P : So, you didn't have much interest in learning because you didn't want to be here in the first place?
 S4 : Yes
 P : But you still have an interest in learning math, right?
 S3 : If I want to learn, I just don't really enjoy it.

The interview excerpt shows that students' interest and motivation in learning are not high. One of the factors that cause learning obstacles is low interest and motivation towards the material. This condition is in line with the findings of (Karlina, Saltifa & Sari, 2021) who stated that the higher the students' interest in learning mathematics, the higher the learning outcomes achieved. In addition to psychological ontogenic obstacles, researchers also observed indications of instrumental student ontogenic obstacles by finding student errors in the process of solving Respondent Ability Test (TKR) questions. This can be observed from the results of student work in Figure 3.

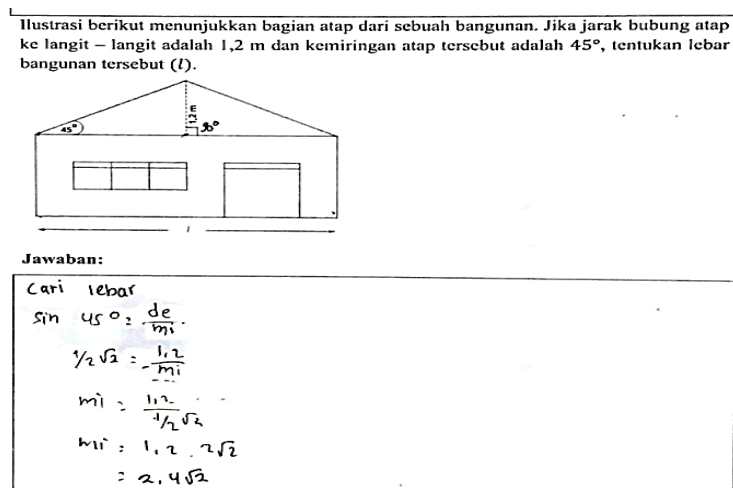


Figure 3. Students Answers with Ontogenic Obstacle (Instrumental)

In Figure 3 above shows that students use the trigonometric ratio $\sin 45^\circ$ in solving the problem, which should use $\tan 45^\circ$. Here are some interview excerpts with students.

- P : Okay, right. What about number four?
- S2 : Number four is looking for the width, so the angle is 45° and the height is 1.2 meters. Then use \sin to find the width. The \sin formula is front per oblique, meaning $\sin 45^\circ$ is half the root of two, so it means this. After that, multiply it by two to make 2.4 roots of two.
- P : Wich width do you think it is?
- S2 : This one (pointing to the picture of the sloping side of the roof)
- P : This one (pointing to the picture of the sloping side of the roof)
- S2 : Yes
- P : Okay, but why do you think it's the width that's skewed?
- S2 : Because usually the width is upwards when drawn
- P : Ohh is that usually the case?
- S2 : Yes, usually so, because in a rectangle, the width is the upright one.

The interview excerpt shows that students have difficulty in understanding the technical concepts that are key to the trigonometric comparison material, especially in identifying the right side to determine the width. This finding is in line with the results of research by (Alfitri et al., 2024) which shows that students' errors in solving problems often stem from technical aspects, namely the lack of understanding and skills in applying concepts appropriately.

In addition to students' psychological and instrumental ontogenic obstacles, there are also indications of conceptual ontogenic obstacles. This can be observed from students' understanding of prerequisite concepts and previous learning experiences expressed through the results of students' completion of the Respondent Ability Test (TKR) in Figure 4 below.

Sebuah pesawat terbang melihat menara pengawas dengan sudut 30° sedangkan tinggi menara pengawas tersebut 200 meter. Berapa jarak antara pesawat terbang tersebut dengan menara pengawas?

Jawaban:

$\cos 30^\circ = \frac{de}{Sa}$
 $\tan 30^\circ = \frac{de}{Sa}$
 $\frac{1}{3}\sqrt{3} = \frac{200}{Sa}$
 $Sa = \frac{200}{\frac{1}{3}\sqrt{3}}$
 $= 200 \cdot 3\sqrt{3}$
 $= 600\sqrt{3} \text{ meter}$

Figure 4. Students Answers with Ontogenic Obstacle (Conceptual)

Figure 4 above shows that students have used trigonometric comparisons correctly, namely by applying $\tan 30^\circ$ to find the distance between the airplane and the watchtower. However, students have not fully understood how to do the root calculation correctly.

This reflects the existence of ontogenic obstacles that are conceptual in nature. Here are some excerpts from interviews with students.

- P : Oh so, let's move on to number seven. How did you do number seven?
- S2 : The plane is from above and then he sees the watchtower from a distance of 200 meters. Then a 30-degree angle is formed, what we are looking for

is the distance between the airplane and the watchtower. If so, it means that we are looking for the side, because the front side is the height of the watchtower 200 meters. Then calculate it to get $600\sqrt{3}$

P : Let's see if the calculation is correct?

S2 : That's right. It's $\frac{1}{3\sqrt{3}}$, so $3\sqrt{3}$, and then it's multiplied by 200 so it becomes $600\sqrt{3}$

Based on the interview excerpt above, it indicates that Subject S2's understanding of root form operation as prerequisite material is not strong. The lack of student mastery of this matter was also revealed by the teacher in the interview. Here are some interview excerpts with the teacher related to this.

P : Before entering the lesson, is there any material that you repeat first as a prerequisite?

G : Yes, I reminded them first about the pythagorean theorem, as well as the root form because it is really used.

P : Yes, ma'am, very used. Regarding this prerequisite material, does everyone understand ma'am?

G : On average, they have, but there are children who are still confused and just add or operate the root form while it can be operated.

P : Where is the confusion mom?

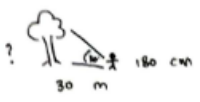
G : Yes, for example, mixing up the pythagorean formulas or like $\sqrt{2}$ can be added with $\sqrt{3}$, or forgetting to rationalize.

Through interviews, the teacher explained that some students still had not mastered the basic concepts that were prerequisite material in learning trigonometric comparisons. This scenario reflects the existence of ontogenic obstacles that are conceptual in nature.

The second epistemological obstacle refers to challenges that arise from students' limited understanding of a specific context or problem (Brousseau, 2002). Based on the TKR results, the researcher suspects that there is an epistemological learning obstacle in trigonometric comparison material. This phenomenon can be observed. The second epistemological obstacle refers to challenges that arise from students' limited understanding of the concepts taught by their teacher. The following are the results of solving TKR problems by students.

5. Adnan merupakan seorang pengamat yang memiliki tinggi 180 cm. Jika Adnan berdiri di depan sebuah pohon besar sejauh 30 meter dan Adnan melihat bagian paling atas pohon tersebut dengan sudut elevasi 30° , maka tentukan tinggi pohon tersebut.

Jawaban:



$$\tan \alpha = \frac{\text{Sisi depan}}{\text{Sisi samping}}$$

$$\tan 30^\circ = \frac{h}{30}$$

$$\frac{1}{\sqrt{3}} = \frac{h}{30}$$

$$\frac{30}{\sqrt{3}} = h$$

$$h = \frac{30}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{30\sqrt{3}}{3} = 10\sqrt{3}$$

Figure 5. Student Answers with Epistemological Obstacle

Figure 5 shows that the student has used trigonometric comparison correctly by applying $\tan 30^\circ$ to calculate the height from the observer's eye to the tip of the tree. However,

students still do not fully understand that this is not the height of the tree as a whole. The following are some interview excerpts with Subject S1.

- P : let's move on to number five.
- S1 : There is a 180 cm tall person standing in a tree 30 m away. So then the angle is 30° , asked to find the height of the tree. That means using $\tan 30^\circ$, the side means the distance is 30 m.
- P : Why tan?
- S1 : The problem is that the height of the tree is the front side, so use tan to know the front side.
- P : Okay, the how?
- S1 : $\tan 30^\circ$ that's one-third the root of three, well then the result is 10 roots of three
- P : So, 10 roots of three is the final result
- S1 : Yes, already
- P : But people look at the eyes, don't they count the height of the person?
- S1 : Does it count?
- P : Do you think it counts from what you know?
- S1 : No, because I've been doing it all this time, its not calculated
- P : Okay, if I ask you if there are two people whose heights are different, for example 120 cm and 180 cm standing in the same place, at the same angle, do you think the height of the tree is the same?
- S1 : Eum, the height will be the same because the calculation uses the distance from the person to the tree and the angle.
- P : Do you usually measure the height of trees from the ground or from people's eyes?
- S1 : From the ground
- P : Okay, so you think it counts from the ground to the top of the tree?
- S1 : Yes, it is.
- P : But if someone looks at a tree with his eyes not parallel to the ground, for example, his height is 180 cm, and then he uses an elevation angle of 30° from his eyes, which part do you calculate?
- S1 : Hmm... yes from the ground too... $\tan 30^\circ$, so just the height from the ground to the top of the tree.
- P : But if you use $\tan 30^\circ$, that's actually calculating from the eye position to the top of the tree, not from the ground. Now, if you just take the result of $\tan 30^\circ$ multiplied by the distance, then you assume it's the height of the tree, don't you forget to add from the ground to the eye?
- S1 : Oh... yes... so what actually counts is from the eyes up?
- P : Yes, that's right. If you don't add the height from the ground to your eyes, you haven't gotten the total height of the tree.
- S1 : I thought I'd get it straight from the ground, because so far I haven't increased the height of the person.

Based on the interview excerpt above, it indicates that Subject S1 still experiences limited understanding and mastery related to trigonometric comparison. Another thing was revealed by Subject S2 in the interview.

- P : What are the questions for daily tests or exams? Is it the same as the example or different?
- S2 : Some are the same, some are different. It's just that I felt the questions were harder than when I was practicing.
- P : Oh, it's more difficult, did you find it difficult when you did it?
- S2 : Yes

Based on the interview excerpt, Subject S2 revealed that there were models of daily test and exam questions that were different from the example questions during class learning. This made Subject S2 have difficulty in solving the problem. According to the results of TKR completion and interviews with students, it shows that students have difficulty solving problems in a form that is different from the examples given during learning by the teacher.

This condition shows that students have not been able to apply the concept of trigonometric comparison in various situations that are different from the learning context they usually experience. This is in line with the theory of situated cognition proposed (Mapa, 2023), which states that knowledge occurs through real activities in a particular environment. That is, if students are only accustomed to solving problems with a certain model, then students will experience difficulties when faced with problems with different models or contexts.

The third, didactical obstacle is learning barriers that arise because of the didactical system or the whole learning process (Brousseau, 2002). The researcher suspects that there is a didactical obstacle, so it needs to be further explored regarding the didactical obstacle. The following are the results of student work that indicated the existence of didactical obstacles.

2. Sebuah segitiga siku – siku memiliki besar sudut 30° , 60° , dan 90° . Jika panjang sisi miring segitiga tersebut adalah 18 cm, tentukan panjang kedua sisi lainnya.

Jawaban :

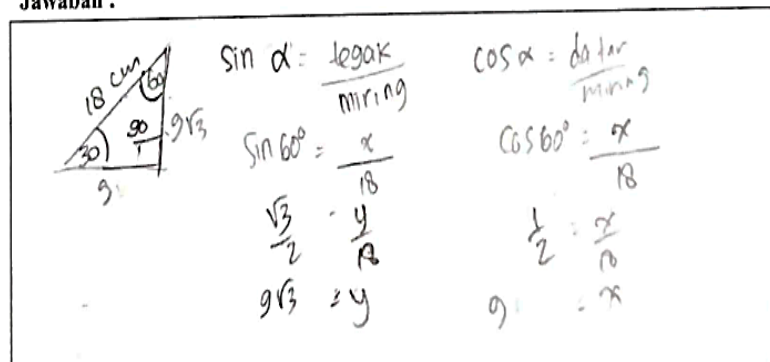


Figure 6. Student answers with Didactical Obstacle

Based on Figure 6 above, it is found that students use the formula $\sin \alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$ directly without paying attention to the angle of reference. This was revealed by the student in the interview.

- P : Okay, on to number two. Number two, can you explain how?
- S4 : Just follow the formula from the notebook.
- P : Is this the formula? The upright per oblique?
- S4 : Yes, it is.
- P : Which side is upright if at? $\sin 60^\circ$?
- S4 : Upright which...is this?

- P : Yes, that's true, but this is 60° right? \sin is actually not upright per oblique, but depends on where we place the angle first. So, we have to know which angle we want to use, because there are still two other angles besides the angle 90° ?
- S4 : Yes, there is still the same 30° and 60°
- P : Yes, there are still two angles. Did you use the upright per oblique from the notes taught in class?
- S4 : Yes, from what is taught in class.
- P : Okay, if you use an upright per tilt if at an angle 90° how?
- S4 : Uemm, confused
- P : If we try to use $\sin 90^\circ$, the answer will be the same as $\sin 60^\circ$? Because the upright side is this, and the hypotenuse is this (pointing to the picture).
- S4 : Heumm yes
- P : Yes, so why do you use $\cos 60^\circ$?
- S4 : Because right, just lacking the flat side, so you can use $\cos 60^\circ$ to find the flat side.

Based on the interview excerpt above, it was found that Subject S4 used the formula $\sin \alpha = \frac{\text{opposite side}}{\text{hypotenuse}}$ which obtained from his class notes. Subject S4 applied this formula directly without considering the reference angle used in the triangle.

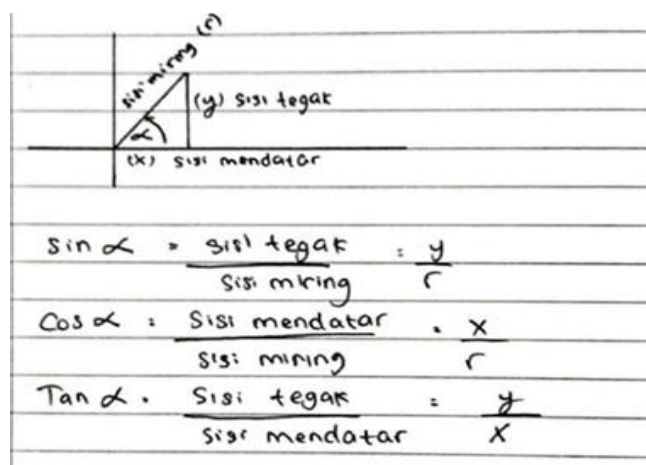


Figure 7. Student Notes During Class Learning

Based on Figure 7 above, this indicates that the student memorized and applied the procedure without truly understanding its meaning within the context of a right-angled triangle. Furthermore, Subject S4 mentioned that he used $\cos 60^\circ$ because the unknown side was the flat side of the triangle. This data reveals that the student did not understand the relative position of the sides in relation to different reference angles. In trigonometry, the choice of function (\sin , \cos , \tan) depends on the position of the side relative to the chosen angle, not just the visual orientation, like "flat side" or "top side".

This error illustrates the presence of a didactical obstacle, which refers to an obstacle that originates from how the material is presented during instruction. The way the content was delivered focused more on rote application of formulas rather than conceptual understanding of the relationship between angles and sides. As a result, the student became confused when required to adapt the formula to different problem contexts.

Another thing is also expressed by students through interviews that reveal shortcomings in learning trigonometric comparisons in class. Here are some interview excerpts with Subjects S1 and S4.

- P : What about the way the teacher teaches in class? Is there anything missing or not?
- S4 : Sometimes it's just too fast to explain, so I haven't understood it yet.
- P : What do you think about the way the teacher teaches in class?
- S1 : I think some times it's a little too fast.
- P : If the next meeting there is a repetition of the previous material?
- S1 : Sometimes it's repeated, but sometimes it's just straightforward.
- P : Do you think that the book should still lack explanation?
- S1 : Yes, a little
- P : Where is it lacking?
- S1 : The understanding of trigonometric comparisons should be clarified again, because I don't think all my friends understand what it means, so I only remember sin, cos, tan.
- P : Oh, so you think it should be added clearly and written what trigonometric comparison is?
- S1 : Yes, because there is no understanding.

Based on the interview excerpts, Subjects S1 and S4 revealed obstacles to learning trigonometric comparisons related to the way the material was delivered by the teacher and the contents of the math book used. Subject S4 said that the teacher's explanation in class tends to take place quickly, so he has not had time to understand the material as a whole before the teacher moves on to the next topic.

Meanwhile, Subject S1 also stated that the teacher sometimes did not repeat the previous material in the next meeting. Subject S1 also discussed the math book used, which did not provide an explanation of the meaning of the concept of trigonometric comparison. In the book, students only obtain the formulas of sin, cos, and tan without a thorough explanation of the meaning of trigonometric comparison and the relationship between sides based on the reference angle.

Through interviews, teachers revealed some of the shortcomings of the didactic system in learning trigonometric comparison. The following are some excerpts of interviews with teachers related to this matter.

- P : Then mom, for the learning time, do you think it is still lacking or is it enough for learning?
- G : Yes, I would say it's enough, but sometimes I get stuck so I rush to finish the material.
- P : So, how did you handle it?
- G : In the end, I tried to teach the important material first, so that the time would be just right, and the others I spoke about briefly.
- P : Regarding the application of trigonometric comparison, did you explain it in class?
- G : Yes, I explained it, but only briefly and not for as long as I remember because there wasn't enough time. Because last semester was full of material
- P : That means it was taught briefly, ma'am, yesterday there were also many students who told me that problem number six that I gave them was difficult. In class, is it taught like this or not?

G : I don't think so, so it's just a glimpse of the angle of elevation and depression. So if it requires more reasoning, it is usually difficult for students to understand because they still have to be guided once.

Based on the interview excerpt above, the teacher said that the available learning time is often insufficient to cover all the material thoroughly. This causes some material, especially related to the application of trigonometric comparisons such as elevation angles and depression angles, to be only taught briefly and not discussed in depth. This condition causes a lack of completeness in the presentation of the material, especially in parts that require higher understanding and reasoning. As a result of this, students lack mastery of problems related to the application of trigonometric comparison material. According to (Brousseau's, 2002) didactic situation theory, the action situations and formulations created by students are still not optimal because their learning is not preceded by problem-solving activities. Meanwhile, learning trigonometry comparisons starts with presenting the problem and continues with solving activities that can encourage the development of student knowledge and create experiences for more meaningful learning (Amedume, Bukari & Mifetu, 2022; Sallah et al., 2023).

CONCLUSION

The meaning of trigonometric ratios that students understand is divided into four categories, which are influenced by the explanations given by the teacher, mathematics textbooks and student notebooks, as well as previous learning experiences. Students have difficulty in choosing the right trigonometric ratio and understanding the context of applied problems. Furthermore, the findings reveal three categories of learning obstacles as proposed by Brousseau, which students encounter: ontogenic obstacles of a psychological nature, indicated by students' low interest and motivation in learning; instrumental obstacles, reflected in the imbalance between students' use of concept images and concept definitions; and conceptual obstacles, due to students' lack of mastery of prerequisite knowledge. In addition, epistemological obstacles were identified, as evidenced by students' difficulties in solving problems presented in various forms, and didactical obstacles, as demonstrated by the teachers' rapid and superficial delivery of the material. These findings suggest that students' learning obstacles are interconnected between internal factors (within the students) and external factors.

REFERENCES

- Alfitri, P. A. A., Kurniati, N., Prabawanto, S., & Yulianti, K. (2024). An Exploration of High School Students' Errors in Solving Trigonometry Problems. *Jurnal Cendekia : Jurnal Pendidikan Matematika*, 8(1), 714–724. <https://doi.org/10.31004/cendekia.v8i1.2109>
- Amedume, L. K., Bukari, H. I., & Mifetu, R. K. (2022). Effect of Problem-Based Learning Approach on Students' Achievement in Trigonometry. *International Journal of Trends in Mathematics Education Research*, 5(4), 348–357. <https://doi.org/10.33122/ijtmer.v5i4.201>
- Brousseau, G. (2002). *Theory of Didactical Situations in Mathematics*. Kluwer Academic Publisher.
- Creswell, J. W. (2013). *Qualitative Inquiry & Research Design Choosing Among Five Approaches* (Second Edition). Sage Publications, Inc.

- Denzin, N. K., & Lincoln, Y. S. (2018). *The SAGE Handbook of Qualitative Research* (Fifth Edition). SAGE Publications, Inc.
- Dhungana, S., Pant, B. P., & Dahal, N. (2023). Students' Experience in Learning Trigonometry in High School Mathematics: A Phenomenological Study. *Mathematics Teaching Research Journal* 184 Golden Fall, 15(4).
- Fajri, N., & Nida, I. (2019). Analisis Kesulitan Siswa Kelas X SMA Negeri 6 Aceh Barat Daya Pada Materi Trigonometri. *Jurnal Ilmiah Pendidikan Matematika AL-QALASADI*, 3(2), 12–22.
- Ghasemi, A., Taghinejad, M., Kabiri, A., & Imani, M. (2011). Ricoeur's Theory of Interpretation: A Method for Understanding Text (Course Text). *World Applied Sciences Journal*, 15(11), 1623–1629.
- Jatisunda, M. G., Suciawati, V., & Nahdi, D. S. (2021). Pythagorean Theorem Concept Image in Junior High School: An Analysis in The Online-Based Learning. *Jurnal Didaktik Matematika*, 8(2), 235–249. <https://doi.org/10.24815/jdm.v8i2.21902>
- Karlina, T., Saltifa, P., & Sari, R. K. (2021). The Correlation Between Learning Interest and Mathematics Achievement in Grade VII SMP PGRI Bengkulu. *Journal of Physics: Conference Series*, 1731(1). <https://doi.org/10.1088/1742-6596/1731/1/012051>
- La Misi, A. A., Saidi, S., & Tamrin Bakar, M. (2023). Learning Obstacle Siswa SMA Negeri 1 Kota Ternate Dalam Memahami Konsep Turunan Fungsi Ditinjau Dari Struktur Materi. *Saintifik@ Jurnal Pendidikan MIPA*, 8(1), 32–42.
- Maharani, A. F., & Prihatnani, E. (2019). Newman's Error Analysis Dalam Geometri Ruang Ditinjau Dari Kecerdasan Visual Siswa SMA. *Journal Cendekia: Jurnal Pendidikan Matematika P-ISSN*, 03(02), 447–461. <http://puspendik.kemdikbud.go.id/hasil-un/>
- Mapa, A. (2023). The Development of Future-proof Learning Materials for Learners with Difficulty in Applying Knowledge. *American Journal of Educational Research*, 11(5), 316–320. <https://doi.org/10.12691/education-11-5-9>
- Nurwahyu, B., Tinungki, G. M., & Mustangin. (2020). Students' Concept Image and Its Impact on Reasoning towards the Concept of the Derivative. *European Journal of Educational Research*, 9(4), 1723–1734. <https://doi.org/10.12973/eu-jer.9.4.1723>
- Pauji, I., Suryadi, D., Bin, M. A., Setambah, B., & Hendriyanto, A. (2023). Learning Obstacle in The Introduction to Number: A Critical Study Within Didactical Design Research Framework. *Indonesian Journal of Science and Mathematics Education*, 3(6), 430–451. <https://doi.org/10.24042/ijsme.v5i1.19792>
- Sallah, E. K., Owusu, A., & Kwame Butakor, P. (2023). Problem Based Learning: Errors Minimization, Conceptual and Procedural Understanding in Trigonometry. *African Journal of Educational Studies in Mathematics and Sciences*, 19(1), 1–20. <https://doi.org/10.4314/ajesms.v19i1.1>
- Sukwantini, S. (2020). Meningkatkan Hasil Belajar Siswa Menggunakan Pembelajaran Kooperatif Problem Based Learning Pada Materi Trigonometri di SMA. *Jurnal Kajian Pembelajaran dan Keilmuan*, 3(2), 173–184.
- Susanti, Nurdayani, A., Raini, Y., Paranita, S., & Ibn Khaldun Bogor, U. (2024). Meningkatkan Hasil Belajar Siswa Pada Materi Perbandingan Trigonometri Pada Segitiga Siku-Siku Menggunakan Media Edukatif Pada Kurikulum Merdeka. *Jurnal Penelitian Pendidikan Matematika*, 8(1), 1–10.

Thasiro, & Ruzai, W. N. A. (2023). Analyzing Trigonometry Learning Obstacles from a Mathematical Literacy Perspective. *Pasundan Journal of Mathematics Education: Jurnal Pendidikan Matematika*, 13(1), 15–29.
<https://doi.org/10.23969/pjme.v13i1.7368>